COORDINATE GEOMETRY

Coordinates of a Point

The Coordinates of a Point

Read the coordinates of a point

Coordinates of a points – are the values of x and y enclosed by the bracket which are used to describe the position of a point in the plane

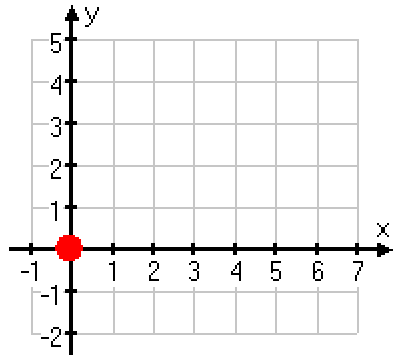
The plane used is called $xy -$ plane and it has two axis; horizontal axis known as $x -$ axis and; vertical axis known as $y -$ axis

A Point Given its Coordinates

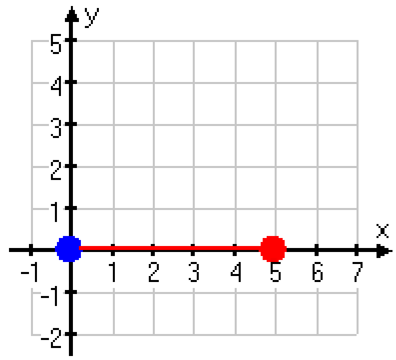
Plot a point given its coordinates

Suppose you were told to locate (5, 2) on the plane. Where would you look? To understand the meaning of (5, 2), you have to know the following rule: The*x*-coordinate (*always*comes first. The first number (the first coordinate) is*always*on the horizontal axis.

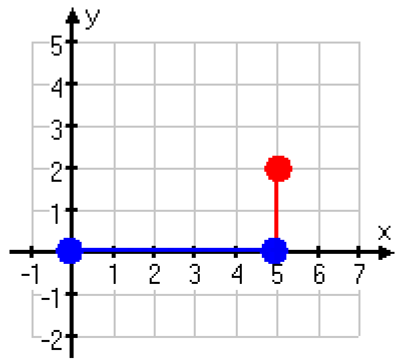
So, for the point $(5, 2)$, you would start at the "origin", the spot where the axes cross:



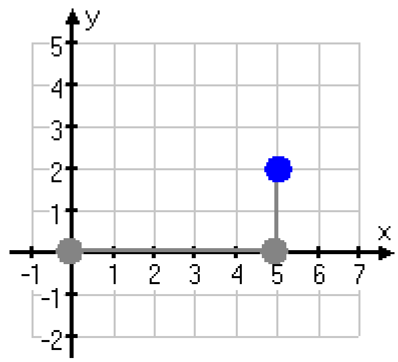
...then count over to "five" on the x-axis:



...then count up to "two", moving parallel to the y-axis:



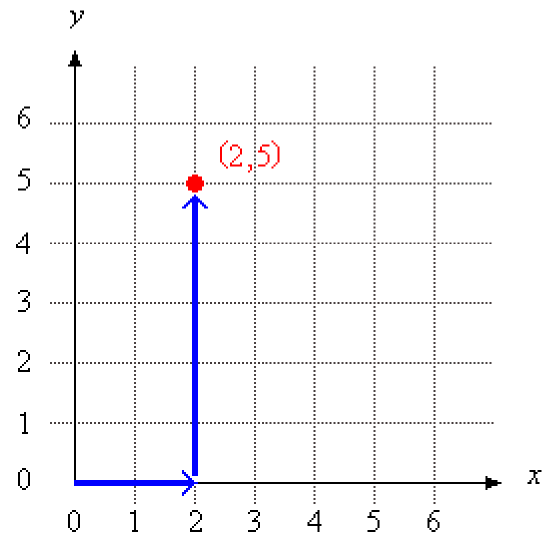
...and then draw in the dot:



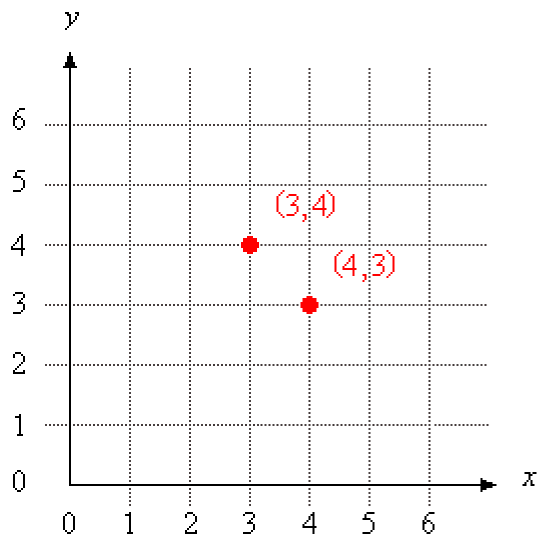
A Point on the Coordinates

Locate a point on the coordinates

The location of (2,5) is shown on the coordinate grid below. The*x*-coordinate is 2. The*y*-coordinate is 5. To locate (2,5), move 2 units to the right on the*x*-axis and 5 units up on the*y*-axis.



The order in which you write*x*- and*y*-coordinates in an ordered pair is very important. Th e*x*-coordinate always comes first, followed by the*y*-coordinate. As you can see in the coordinate grid below, the ordered pairs (3,4) and (4,3) refer to two different points!



Gradient (Slope) of a Line

The Gradient of a Line Given Two Points

Calculate the gradient of a line given two points

Gradient or slope of a line – is defined as the measure of steepness of the line. When using coordinates, gradient is defined as change in y to the change in x.

$$\text{Gradient} = \frac{\text{change in } y}{\text{change in } x}$$

Consider two points A (x_1, y_1)and (B x_2, y_2), the slope between the two points is given by:

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

OR

$$\text{Gradient} = \frac{y_1 - y_2}{x_1 - x_2}$$

Example 1

Find the gradient of the lines joining:

a. (5, 1) and (2,−2)

b. (4,−2) and (−1, 0)

c. (−2,−3) and (−4,−7)

Solution

(a) $(5,1)$ and $(2,-2)$

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-1}{2-5} = \frac{-3}{-3} = 1$$

(b) $(4,-2)$ and $(-1,0)$

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - -2}{-1-5} = \frac{2}{-6} = -\frac{1}{3}$$

(c) $(-2,-3)$ and $(-4,-7)$

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7 - \ -3}{-4 - -2} = \frac{-7+3}{-4+2} = \frac{-4}{-2} = 2$$

Example 2

a. The line joining (2,−3) and (k, 5) has gradient −2. Find k

b. Find the value of m if the line joining the points (−5,−3) and (6,m) has a slope of½

Solution

(a) Given $(2,-3)$ and $(k,5)$

gradient $= \frac{y_2-y_1}{x_2-x_1}$

$$-2 = \frac{5 - \ -3}{k-2}$$
$$-2(k-2) = 5+3$$
$$-2k+4 = 8$$
$$-2k = 8-4$$
$$-2k = 4$$
$$k = \frac{4}{-2} = -2$$

$\therefore$ The value of k is -2

(b) Given $(-5,-3)$ and $(6,m)$

$$\text{gradient} = \frac{y_2-y_1}{x_2-x_1}$$
$$\frac{1}{2} = \frac{m - \ -3}{6 - -5}$$

$$\frac{1}{2} = \frac{m+3}{6+5}$$

$$\frac{1}{2} = \frac{m+3}{11}$$

$$2(m+3) = 11$$
$$2m+6 = 11$$
$$2m = 11-6$$
$$2m = 5$$
$$m = \frac{5}{2}$$

The value of k is $\frac{5}{2}$

The Equations of a Line Given the Coordinates of Two Points on a Line

Find the equations of a line given the coordinates of two points on a line

The equation of a straight line can be determined if one of the following is given:-

- The gradient and the y − intercept (at x = 0) or x − intercept (at y=0)
- The gradient and a point on the line
- Since only one point is given, then

$$\text{gradient} = \frac{y - y_1}{x - x_1}$$

- Two points on the line

Example 3

Find the equation of the line with the following

a. Gradient 2 and y − intercept −4

b. Gradient $-2/3$ and passing through the point (2, 4)

c. Passing through the points (3, 4) and (4, 5)

Solution

(a) Given $m = 2$ and $c = -4$

$$y = mx + c$$
$$y = 2x - 4$$

(b) Recall

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$-\frac{2}{3} = \frac{y - 4}{x - 2}$$

$$-2(x - 2) = 3(y - 4)$$

$$-2x + 4 = 3y - 12$$

$$-2x + 4 - 3y + 12 = 0$$

$$-2x - 3y + 16 = 0$$

Divide by the negative sign, (−), throughout the equation

∴The equation of the line is $2x + 3y - 16 = 0$

(c) Recall

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{5-4}{4-3} = \frac{1}{1} = 1$$

Then

$$\text{gradient} = \frac{y - y_1}{x - x_1}$$

$$1 = \frac{y-4}{x-3}$$

$$x - 3 = y - 4$$

$$x - 3 - y + 4 = 0$$

$$x - y + 1 = 0$$

∴The equation of the line is $x - y + 1 = 0$

The equation of a line can be expressed in two forms

a. $ax + by + c = 0$ and

b. $y = mx + c$

Consider the equation of the form $y = mx + c$

m = Gradient of the line

Example 4

Find the gradient of the following lines

a. $2y = 5x + 1$

b. $2x + 3y = 5$

c. $x + y = 3$

Solution

(a) Express in the form of $y = mx + c$

Divide by both sides

$$y = \frac{5x+1}{2} = \frac{5}{2}x + \frac{1}{2}$$
$$y = \frac{5}{2}x + \frac{1}{2}$$

$\therefore$ Gradient $= \frac{5}{2}$

(b) Express in the form of $y = mx + c$

Divide by both sides

$$2x + 3y = 5$$
$$3y = 5 - 2x$$
$$3y = -2x + 5$$
$$y = \frac{-2x+5}{3} = -\frac{2}{3}x + \frac{5}{3}$$
$$y = -\frac{2}{3}x + \frac{5}{3}$$

$\therefore$ Gradient $= \frac{5}{2}$

(c) $x + y = 3$

Express in the form of $y = mx + c$

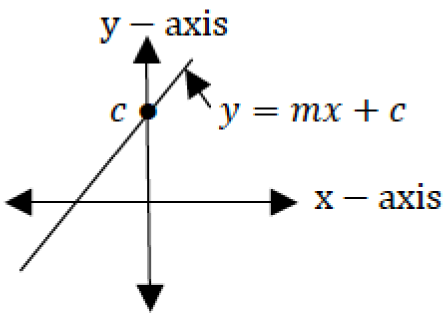
$$y = 3 - x$$
$$y = -x + 3$$

$\therefore$ Gradient $= -1$

Intercepts

The line of the form $y = mx + c$, crosses the $y - axis$ when $x = 0$ and also crosses $x - axis$ when $y = 0$

See the figure below



Therefore

a. to get x − intercept, let $y = 0$ and

b. to get y − intercept, let $x = 0$

From the line, $y = mx + c$

y − intercept, let $x = 0$

$y = m\ 0 + c = 0 + c = c$

y − intercept = c

Therefore, in the equation of the form $y = mx + c$, m is the gradient and c is the y − intercept

Example 5

Find the y − intercepts of the following lines

(a) $y = 3x + 5$

(b) $y = -\frac{1}{2}x + \frac{2}{3}$

(c) $3y = 2x + 1$

Solution

(a) $y = 3x + 5$

Compare with $y = mx + c$

$$y - \text{intercept} = c = 5$$

$\therefore y - \text{intercept is } 5$

(b) $y = -\frac{1}{2}x + \frac{2}{3}$

$$y - \text{intercept} = \frac{2}{3}$$

(c) $3y = 2x + 1$

Express in the form of $y = mx + c$

Divide by 3 both sides

$$y = \frac{2x+1}{3} = \frac{2}{3}x + \frac{1}{3}$$

$$y = \frac{2}{3}x + \frac{1}{3}$$

$$y - \text{intercept} = \frac{1}{3}$$

Graphs of Linear Equations

The Table of Value

Form the table of value

The graph of a straight line can be drawn by using two methods:

a. By using intercepts

b. By using the table of values

Example 6

Sketch the graph of $y = 2x - 1$

Solution

By using intercepts

$y -$ intercept, let $x = 0$

$$y = 2(0) - 1$$
$$y = 0 - 1$$
$$y = -1$$

$x -$ intercept, let $y = 0$

$$0 = 2x - 1$$
$$2x - 1 = 0$$
$$2x = 1$$
$$x = \frac{1}{2}$$

The coordinates are $\left(\frac{1}{2}, 0\right)$ and (0, -1)

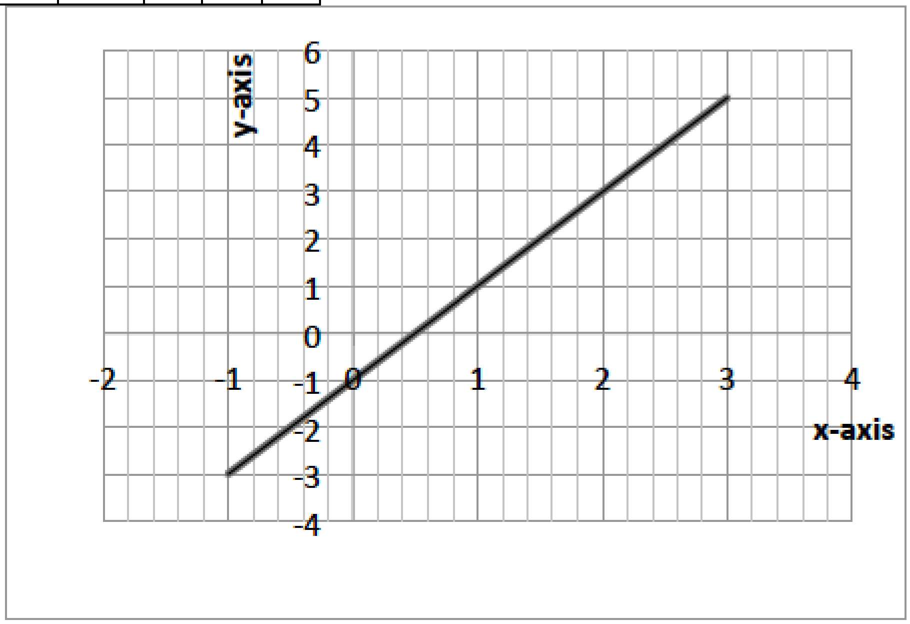
Then show the straight line through the point $\left(\frac{1}{2}, 0\right)$ and (0, -1) on the xy −plane.

The Graph of a Linear Equation

Draw the graph of a linear equation

By using the table of values

x	-1	0	1	2	3
y	-3	-1	1	3	5



Simultaneous Equations

Linear Simultaneous Equations Graphically

Solve linear simultaneous equations graphically

Use the intercepts to plot the straight lines of the simultaneous equations. The point where the two lines cross each other is the solution to the simultaneous equations

Example 7

Solve the following simultaneous equations by graphical method

$$\begin{cases} x + y = 4 \\ 2x - y = 2 \end{cases}$$

Solution

Consider: $x + y = 4$

If $x = 0$, $0 + y = 4$ $y = 4$

If $y = 0$, $x + 0 = 4$ $x = 4$

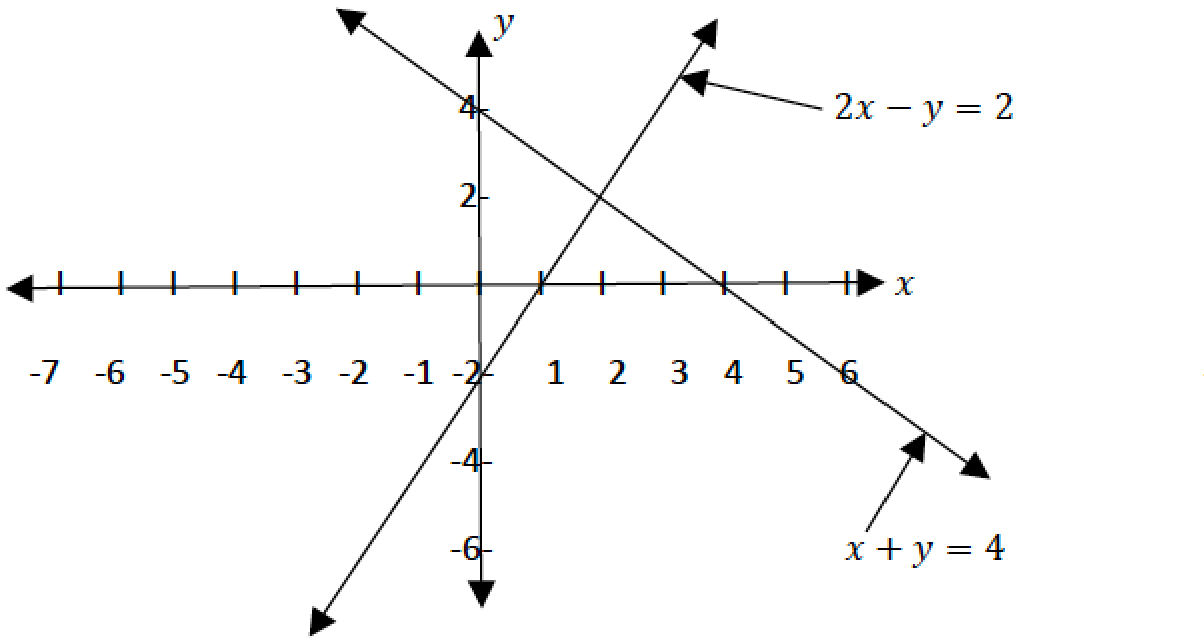
Draw a straight line through the points 0, 4 and 4, 0 on the $xy-$ plane

Consider: $2x - y = 2$

If $x = 0$, $0 - y = 2$ $y = -2$

If $y = 0$, $2x - 0 = 2$ $x = 1$

Draw a straight line through the points (0,−2) and (1, 0) on the $xy-$ plane



From the graph above the two lines meet at the point 2, 2 , therefore $x = 2$ *and* $y = 2$